Barker College

Student Number: ..............................

# 2010

## YEAR 12
## TRIAL HSC
## EXAMINATION

# MATHEMATICS

**Staff Involved:**

**THURSDAY 5TH AUGUST**

- TRW
- GIC
- VAB
- GPF*
- WMD*
- LJP
- JGD
- KJL
- AJD

**130 copies**

## General Instructions

- **Reading time – 5 minutes**
- **Working time – 3 hours**
- **Write using blue or black pen**
- **Write your Barker Student Number on all pages of your answers**
- **Board-approved calculators may be used**
- **A Table of Standard Integrals is provided at the back of this paper which may be detached for your use**
- **ALL necessary working MUST be shown in every question**
- **Marks may be deducted for careless or badly arranged working**

## Total marks - 120

- **Attempt Questions 1 - 10**
- **All questions are of equal value**
- **BEGIN your answer to EACH QUESTION on a NEW PIECE of the separate lined paper**
- **Write only on ONE SIDE of the separate lined paper**

-**BLANK PAGE -**

**Total marks - 120**
**Attempt Questions 1 - 10**
**All questions are of equal value**

**Answer each question on a separate A4 lined sheet of paper.**

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**Marks**

**Question 1** (12 marks) **[START A NEW PAGE]**

(a) Evaluate, to 3 significant figures, $\dfrac{(-2.4)^2}{\sqrt{2\pi - 5}}$. **2**

(b) Simplify fully $\dfrac{16a^3 - 54b^3}{4a^2 + 6ab + 9b^2}$. **2**

(c) Solve for $x$: $|2x - 5| < 8$. **2**

(d) Write down the domain of the function $y = \dfrac{1}{\sqrt{9 - x}}$. **1**

(e) Solve for $x$: $9^x - 7(3^x) - 18 = 0$. **3**

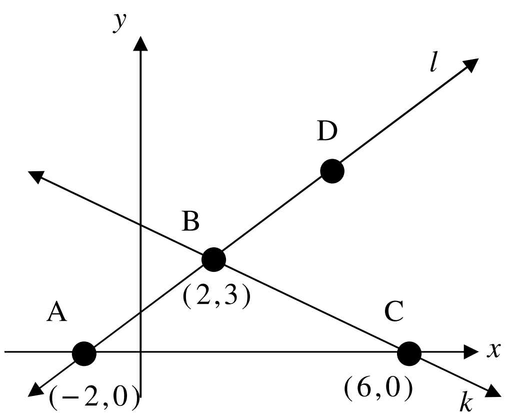
(f) Determine whether $f(x) = \dfrac{x}{x^2 - 3}$ is odd, even or neither, justifying your answer with appropriate working. **2**

**End of Question 1**

**Marks**

**Question 2** (12 marks) **[START A NEW PAGE]**

The diagram shows the lines $l$ and $k$:




NOT TO SCALE.

Redraw this diagram in your answer booklet.

(i) Calculate the gradient of the line $l$. **1**

(ii) Calculate the length of AB. **1**

(iii) Find in **general form**, the equation of the line $l$. **2**

(iv) Calculate the angle of inclination of the line $l$ (to the nearest degree). **1**

(v) D is a point on the line $l$, such that AD and CD are perpendicular.
Find the coordinates of D. **4**

(vi) Show the area of the $\Delta ABC$ is 12 square units. **1**

(vii) Find $\angle ABC$ (to the nearest degree). **2**

**End of Question 2**

**Marks**

**Question 3** (12 marks) **[START A NEW PAGE]**

(a) Differentiate the following expressions with respect to *x*:

(i) $\sqrt{x}\ln x$ **2**

(ii) $\tan\left(\frac{\pi}{2} - x^2\right)$ **2**

(b) Find $\frac{d}{dx}\left[\frac{x}{e^{x^2}}\right]$ in simplest form. **3**

(c) Find the values of $k$ for which $x^2 + (k+3)x - k = 0$ has real roots. **3**

(d) If $\alpha$ and $\beta$ are the roots of $2x^2 - 5x + 1 = 0$, **2**
find the value of $\alpha^2 + \beta^2$.

**End of Question 3**

**Marks**

**Question 4** (12 marks) **[START A NEW PAGE]**

(a) For the function with the equation $y = 3\sin(\frac{x}{2} + \frac{\pi}{4})$

(i) State the amplitude of the function. **1**

(ii) State the period of the function. **1**

(iii) Sketch the graph of the function over the domain $0 \le x \le 2\pi$. **2**

(b) Solve the equation $4\sin x = 3\operatorname{cosec} x$ for $0° \le x \le 360°$. **3**

(c) Prove $\frac{\cos\alpha}{1+\sin\alpha} = \sec\alpha(1-\sin\alpha)$. **3**

(d) Find the 32nd term of the series $(-6) + (-2) + 2 + ...$ **1**

(e) Find the limiting sum of the series $108 + 36 + 12 + ...$ **1**

**End of Question 4**

**Marks**

**Question 5** (12 marks) **[START A NEW PAGE]**

(a) Find $\int \frac{dx}{\sqrt[3]{x^2}}$. **2**

(b) Evaluate $\int_2^{\sqrt{7}} \frac{x}{x^2 - 3}\, dx$. **3**

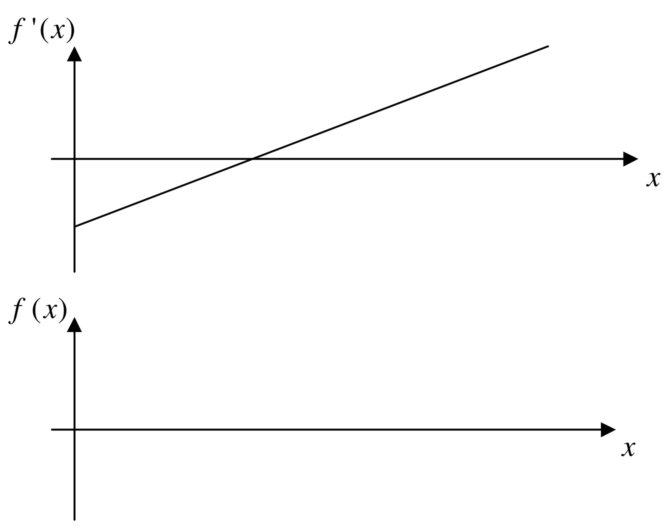
(c) (i) **In your answer booklet,** copy and complete the table below for the function $f(x) = e^x$, giving your answers to 3 decimal places where necessary. **1**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | | | |

(ii) Using Simpson's rule with 3 function values, find an approximation to $\int_0^2 e^x dx$. **2**

(d) Find the volume of the solid of revolution formed by rotating the line $y = -2x$ between $x = 1$ and $x = 5$ about the $x$-axis. **3**

(e) **In your answer booklet**, copy the diagrams below. Use the graph of $y = f'(x)$ to complete a possible graph of $y = f(x)$. **1**

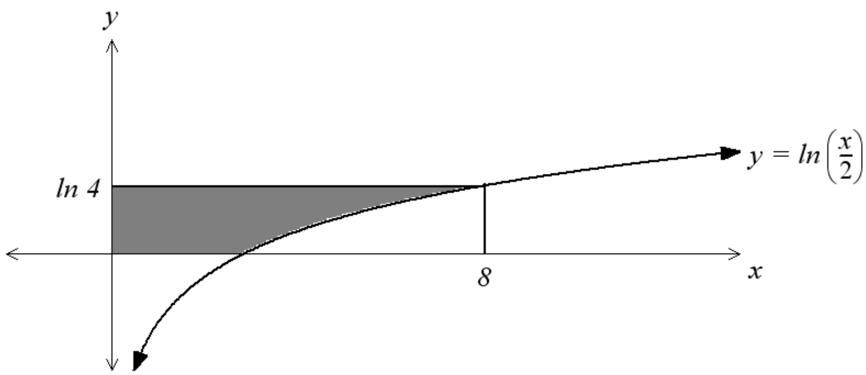



**End of Question 5**

**Marks**

**Question 6** (12 marks) **[START A NEW PAGE]**

(a)




The diagram shows the area bounded by the graph $y = ln\left(\frac{x}{2}\right)$,

the co-ordinates axes and the line $y = ln\ 4$.

Find the area of the shaded region. **3**

(b) The curve $y = f(x)$ has a gradient function of $\frac{dy}{dx} = (1-x)^3$.
The curve passes through the point $(-1, 1)$.
Find the equation of the curve. **3**

(c) If $\cos\beta = \frac{2}{5}$ and $\sin\beta < 0$, find the exact value of $\tan\beta$. **2**

(d) Find the exact value of $\text{cosec}\left(\frac{5\pi}{3}\right)$. **2**

(e) Express $\frac{2}{\sqrt{5}-1} - \frac{3}{\sqrt{5}+1}$ in its simplest form. **2**

**End of Question 6**

**Marks**

**Question 7** (12 marks) **[START A NEW PAGE]**

(a) A town's population was recorded at the start of 2004. The population, P, $t$ years later is given by the exponential equation $P = 120000e^{-0.05t}$.

(i) What was the initial population of the town at the start of 2004? **1**

(ii) Find the time in years it will take the initial population to halve. **2**

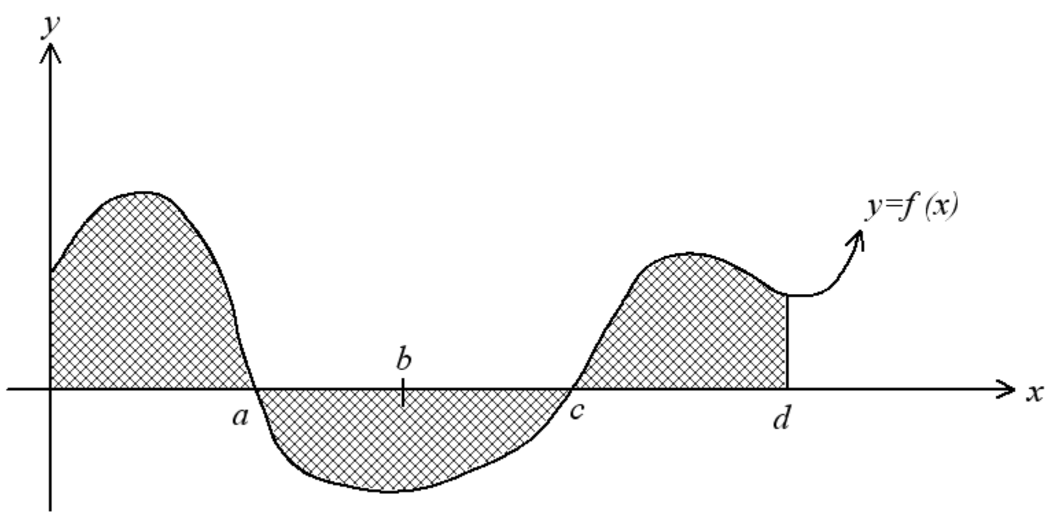
(iii) At what rate is the population changing at the start of 2010? **2**

(b) (i) Show the locus of a point P ( $x$ , $y$ ) which moves so that PA = 2PB, where A is the point $(-2, 4)$ and B is $(4, 1)$ is:

$x^2 + y^2 - 12x + 16 = 0$ **3**

(ii) Find the centre and radius of the circle defined in (i). **2**

(c) Write an expression for the shaded area shown in the diagram below. **2**

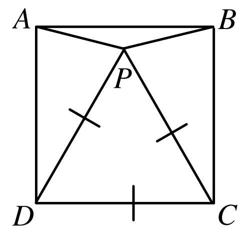



**End of Question 7**

**Marks**

**Question 8** (12 marks) **[START A NEW PAGE]**

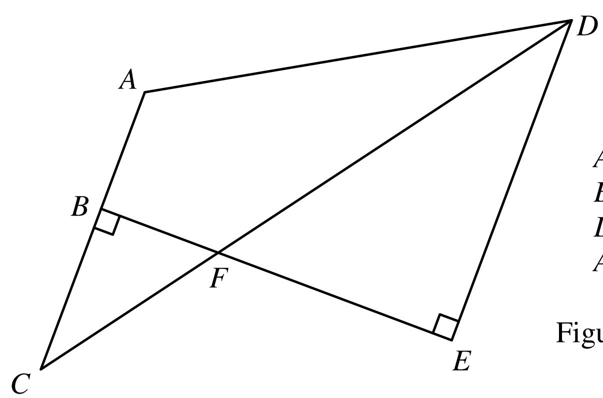
(a) P is a point inside a square ABCD such that triangle PDC is equilateral.
Prove that:




(i) $\Delta APD \equiv \Delta BPC$. **3**

(ii) $\Delta APB$ is isosceles. **1**

(b) Copy the following diagram onto your answer sheet.




$AB = 5$ cm
$BC = 6$ cm
$DE = 10$ cm
$AD = 13$ cm

Figure not to scale

(i) Prove $\Delta BFC \parallel\!| \Delta EFD$. **2**

(ii) Find the length of DF. **3**

(c) Sketch the parabola $(y-1)^2 = -8(x+2)$, clearly showing the:

(i) coordinates of the vertex **1**

(ii) coordinates of the focus **1**

(iii) equation of the directrix **1**

**End of Question 8**

**Marks**

**Question 9** (12 marks) **[START A NEW PAGE]**

Two particles A and B move along the $x$-axis, both starting when $t = 0$.
The displacement of particles A and B is given by $x = t + 12 - t^2$ and $x = t^2 - 4t$ respectively. In both cases $x$ is the displacement of the particle from O in metres and $t$ is measured in seconds.

(i) Find when and where particle A is stationary. **2**

(ii) On the same diagram, sketch each particle's displacement graph, showing all intercepts. **3**

(iii) Show that the distance, D, between the two particles during $0 \le t \le 4$, is given by $D = 5t + 12 - 2t^2$. **1**

(iv) During the first 4 seconds, when are the particles furthest apart? **2**

(v) Find the time when both particles have the same velocity. **2**

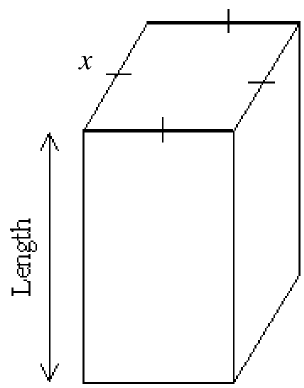
(vi) Make a statement about the accelerations of the particles, being sure to justify your statement with the aid of mathematical evidence. **2**

**End of Question 9**

**Marks**

**Question 10** (12 marks) **[START A NEW PAGE]**

(a) A block of wood is in the shape of a square-based prism.
The **sum** of the length of the block and the perimeter of the base is 12 cm.




(i) Show that $V = 12x^2 - 4x^3$, where $V$ is the volume of the block. **2**

(ii) What is the volume of the largest block? **3**

(b) (i) Greg borrows $400000 is order to buy an apartment. The interest rate is 6% p.a. reducible and the loan is to be repaid in equal monthly repayments of $\$M$ over 25 years with the interest calculated monthly. Let $\$A_n$ be the amount owing after the $n$th repayment.

($\alpha$) Write down expressions for $\$A_1$ and $\$A_2$, the amounts owing after the first and second repayments have been made respectively. **1**

($\beta$) Show that the amount of each monthly repayment is $2577.21 (correct to the nearest cent). **2**

(ii) After 5 years (i.e. 60 repayments) the interest rate rises to 9% p.a. Find the new monthly repayment $\$N$, correct to the nearest cent. (Assume that the period of the loan is still 25 years). **3**

(iii) How much extra does Greg repay over the life of the loan as a result of the 3% p.a. interest rate rise? **1**

**End of Question 10**

**End of Paper**

# STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \; ; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

# 2010 Mathematics Trial Solutions

**1** (a) $\frac{(-2.4)^2}{\sqrt{2\pi - 5}} = 5.084845882$

$= \underline{5.08}$ (3 s.f.)

(b) $\frac{16a^3 - 54b^3}{4a^2+6ab+9b^2}$

$= \frac{2(8a^3 - 27b^3)}{4a^2+6ab+9b^2}$

$= \frac{2(2a-3b)(4a^2+6ab+9b^2)}{4a^2+6ab+9b^2}$

$= \underline{\underline{2(2a-3b)}}$

(c) $|2x-5| < 8$

$\therefore -8 < 2x-5 < 8$

$\therefore -3 < 2x < 13$

$\therefore \underline{\underline{-\frac{3}{2} < x < \frac{13}{2}}}$

(d) $9 - x > 0$

$\Rightarrow$ Domain : $x \underline{< 9}$

(e) $9^x - 7(3^x) - 18 = 0$

$(3^x)^2 - 7(3^x) - 18 = 0$

let $u = 3^x$ :

$u^2 - 7u - 18 = 0$

$(u-9)(u+2) = 0$

$u = 9$ or $u = -2$

$\therefore 3^x = 9$ or $3^x = -2$ ← impossible.

$\therefore 3^x = 3^2$

$\therefore \underline{\underline{x = 2}}$

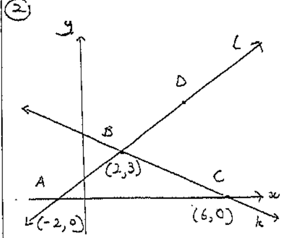
(f) $f(x) = \frac{x}{x^2-3}$

$f(-x) = \frac{-x}{(-x)^2-3} = \frac{-x}{x^2-3}$

$= -f(x)$

$\Rightarrow f(x)$ is an odd function.

**2**




(i) grad. of $l = \frac{3-0}{2--2} = \underline{\underline{\frac{3}{4}}}$

(ii) $d_{AB} = \sqrt{(2--2)^2 + (3-0)^2} = \underline{\underline{5}}$ units

(iii) $y - 3 = \frac{3}{4}(x-2)$

$4y - 12 = 3x - 6$

$\therefore \underline{\underline{3x - 4y + 6 = 0}}$ ← eq'n of $l$.

(iv) $\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(\frac{3}{4}) = 36.86989765°$

$= \underline{\underline{37°}}$ (n.deg.)

(v) Grad. of CD $= -\frac{4}{3}$

Eq'n of CD : $y - 0 = -\frac{4}{3}(x-6)$

$3y = -4x + 24$

$4x + 3y - 24 = 0$ .... (1)

Now $l$ is $3x - 4y + 6 = 0$ .... (2)

(1) × 3 ⇒ $12x + 9y - 72 = 0$ ... (1a)

(2) × 4 ⇒ $12x - 16y + 24 = 0$ ... (2a)

(1a) − (2a) ⇒ $25y - 96 = 0$

$y = \frac{96}{25}$

sub $y = \frac{96}{25}$ into (2):

$3x - 4 \times \frac{96}{25} + 6 = 0$

$x = \frac{78}{25}$

$\therefore$ D is $\underline{\underline{(\frac{78}{25}, \frac{96}{25})}}$

**2** (continued)

(vi) Area $\triangle ABC = \frac{1}{2} \times (6--2) \times 3$

$= \underline{\underline{12u^2}}$

(vii) Method 1

Area $\triangle ABC = \frac{1}{2} \times AB \times BC \times \sin\angle ABC$

$\therefore 12 = \frac{1}{2} \times 5 \times 5 \times \sin\angle ABC$

$\therefore \sin\angle ABC = \frac{24}{25}$

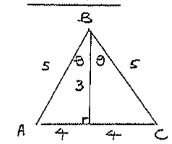
$\angle ABC$ is obtuse because $AB^2 + BC^2 < AC^2$.

$\therefore \angle ABC = 180° - \sin^{-1}(\frac{24}{25})$

$= 106.2602047$

$= \underline{\underline{106°}}$ (n.deg.)

Method 2




$\tan\theta = \frac{4}{3}$

$\theta = 53.130....$

$\therefore \angle ABC = 106.2602047$

$= \underline{\underline{106°}}$ (n.deg.)

Method 3

$\cos\angle ABC = \frac{5^2+5^2-8^2}{2 \times 5 \times 5}$

$= -\frac{7}{25}$

$\therefore \angle ABC = \cos^{-1}(-\frac{7}{25})$

$= 106.2602047$

$= \underline{106°}$ (n.deg.)

Method 4

$\tan\theta = \left|\frac{\frac{3}{4} - -\frac{3}{4}}{1 - \frac{9}{16}}\right| = \frac{24}{7}$

$\angle ABC = 180° - \tan^{-1}(\frac{24}{7})$

$= 106.2602047 = 106°$ (n.deg.)

**3** (a)(i) $\frac{d}{dx}(x^{\frac{1}{2}} \ln x)$

$= x^{\frac{1}{2}} \times \frac{1}{x} + \ln x \times \frac{1}{2}x^{-\frac{1}{2}}$

$= x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} \ln x$

$= \underline{\underline{\frac{1}{\sqrt{x}}(1 + \frac{1}{2}\ln x)}}$

(ii) $\frac{d}{dx}\left(\tan(\frac{\pi}{2} - x^2)\right)$

$= \sec^2(\frac{\pi}{2} - x^2) \times -2x$

$= \underline{\underline{-2x\sec^2(\frac{\pi}{2} - x^2)}}$

(b) $\frac{d}{dx}\left[\frac{x}{e^{x^2}}\right]$

$= \frac{e^{x^2} \times 1 - x \times e^{x^2} \times 2x}{(e^{x^2})^2}$

$= \frac{e^{x^2}(1 - 2x^2)}{(e^{x^2})^2}$

$= \underline{\underline{\frac{1-2x^2}{e^{x^2}}}}$

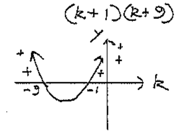
(c) $\Delta = (k+3)^2 - 4(1)(-k)$

$= k^2 + 6k + 9 + 4k$

$= k^2 + 10k + 9$

for real roots $k^2 + 10k + 9 \geq 0$

$(k+1)(k+9) \geq 0$




$\therefore \underline{\underline{k \leq -9 \text{ or } k \geq -1}}$

(d) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\frac{5}{2})^2 - 2(\frac{1}{2})$

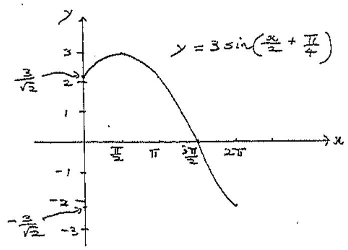
$= \underline{\underline{\frac{21}{4}}}$

④ (a) (i) Amp. = 3 units

(ii) Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

(iii)

| $x$ | 0 | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|---|---|
| $y$ | $\frac{3}{\sqrt{2}}$ | 3 | $\frac{3}{\sqrt{2}}$ | 0 | $-\frac{3}{\sqrt{2}}$ |




(b) $4\sin x = 3\,\text{cosec}\,x$

$4\sin x = \frac{3}{\sin x}$

$\therefore \sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

Basic angle = $60^\circ$

$x = 60^\circ, 180^\circ - 60^\circ, 180^\circ + 60^\circ, 360^\circ - 60^\circ$

$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

(c) Prove $\frac{\cos\alpha}{1+\sin\alpha} = \sec\alpha(1-\sin\alpha)$

LHS = $\frac{\cos\alpha}{1+\sin\alpha} \times \frac{1-\sin\alpha}{1-\sin\alpha}$

$= \frac{\cos\alpha(1-\sin\alpha)}{1-\sin^2\alpha}$

$= \frac{\cos\alpha(1-\sin\alpha)}{\cos^2\alpha}$

$= \frac{1-\sin\alpha}{\cos\alpha}$

$= \sec\alpha(1-\sin\alpha)$

= RHS

(d) $(-6) + (-2) + 2 + \dots$

$T_2 - T_1 = 4 = T_3 - T_2 \Rightarrow$ AP with $d = 4, a = -6$

$T_{32} = a + 31d$

$= -6 + 31\times 4$

$= 118$

(e) $108 + 36 + 12$

$r = \frac{36}{108} = \frac{12}{36} = \frac{1}{3}$

$S_\infty = \frac{108}{1-\frac{1}{3}} = \frac{108}{\frac{2}{3}} = 162$

⑤ (a) $\int \frac{dx}{\sqrt[3]{x^2}} = \int \frac{dx}{x^{\frac{2}{3}}}$

$= \int x^{-\frac{2}{3}}\,dx$

$= \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$

$= 3\sqrt[3]{x} + c$

(b) $\int_2^{\sqrt{7}} \frac{x\,dx}{x^2-3} = \frac{1}{2}\int_2^{\sqrt{7}} \frac{2x}{x^2-3}\,dx$

$= \frac{1}{2}\left[\ln(x^2-3)\right]_2^{\sqrt{7}}$

$= \frac{1}{2}\left\{(\ln 4) - (\ln 1)\right\}$

$= \frac{1}{2}\ln 4 = \ln 4^{\frac{1}{2}}$

$= \ln 2$

(c) (i)

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | 2.718 | 7.389 |

(ii) $\int_0^2 e^x\,dx \doteqdot \frac{h}{3}\left\{y_1 + 4y_2 + y_3\right\}$

$= \frac{1}{3}\left\{1 + 4\times 2.718 + 7.389\right\}$

$= 6.420333333$

(d) $V = \pi\int_1^5 (-2x)^2\,dx$

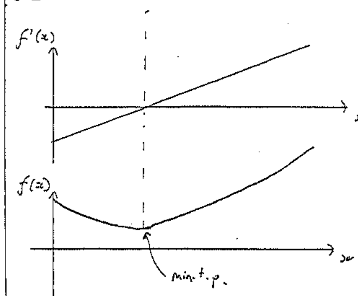
$= 4\pi\int_1^5 x^2\,dx$

$= 4\pi\left[\frac{x^3}{3}\right]_1^5$

$= \frac{4\pi}{3}\left\{125 - 1\right\}$

$= \frac{496\pi}{3}\ u^3$

(e)



⑥

(a) $y = \log_e\left(\frac{x}{2}\right)$

$\therefore e^y = \frac{x}{2}$

$\therefore x = 2e^y$

Req'd Area $= 2\int_0^{\ln 4} e^y \, dy$

$= 2\left[e^y\right]_0^{\ln 4}$

$= 2\left\{e^{\ln 4} - e^0\right\}$

$= 2\{4 - 1\}$

$= \underline{6 u^2}$

(b) $\frac{dy}{dx} = (1-x)^3$

$y = \frac{(1-x)^4}{(-1)\times 4} + c$

sub in $(-1, 1)$:

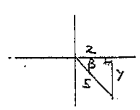
$1 = \frac{(1+1)^4}{-4} + c$

$1 = -4 + c$

$c = 5$

$\therefore y = \underline{-\frac{1}{4}(1-x)^4 + 5}$

(c)




$y^2 + 2^2 = 5^2$

$y^2 = 21$

$y = -\sqrt{21} \quad (y<0)$

$\therefore \tan\beta = \underline{-\frac{\sqrt{21}}{2}}$

(d) $\operatorname{cosec}\left(\frac{5\pi}{3}\right)$

$= \frac{1}{\sin\left(\frac{5\pi}{3}\right)}$

$= \frac{1}{-\sin\frac{\pi}{3}}$

$= \frac{1}{-\frac{\sqrt{3}}{2}}$

$= \underline{-\frac{2}{\sqrt{3}}}$

(e) $\frac{2}{\sqrt{5}-1} - \frac{3}{\sqrt{5}+1}$

$= \frac{2(\sqrt{5}+1) - 3(\sqrt{5}-1)}{(\sqrt{5}-1)(\sqrt{5}+1)}$

$= \frac{2\sqrt{5}+2-3\sqrt{5}+3}{5-1}$

$= \underline{\frac{5-\sqrt{5}}{4}}$

⑦

(a) (i) when $t=0$, $P = \underline{120\,000}$

(ii) when $P = 60\,000$,

$60\,000 = 120\,000 e^{-0.05t}$

$\therefore e^{-0.05t} = \frac{1}{2}$

$\therefore -0.05t = \ln\left(\frac{1}{2}\right)$

$\therefore t = \frac{\ln\left(\frac{1}{2}\right)}{-0.05}$

$= 13.86294361$

$= \underline{13.9 \text{ years}}$ (1d.p.)

(iii) $\frac{dP}{dt} = -0.05 \times 120\,000 e^{-0.05t}$

$= -6000 e^{-0.05t}$

when $t = 6$, $\frac{dP}{dt} = -6000 e^{-0.05\times 6}$

$= -6000 e^{-0.3}$

$= -4444.909324$

Req'd rate $\doteqdot \underline{-4445}$ p.a.

(b) (i) $PA = 2PB$

$\Rightarrow \sqrt{(x+2)^2 + (y-4)^2} = 2\sqrt{(x-4)^2 + (y-1)^2}$

$\therefore (x+2)^2 + (y-4)^2 = 4\left\{(x-4)^2 + (y-1)^2\right\}$

$\therefore x^2 + 4x + 4 + y^2 - 8y + 16$

$= 4\left\{x^2 - 8x + 16 + y^2 - 2y + 1\right\}$

$\therefore x^2 + y^2 + 4x - 8y + 20$

$= 4x^2 + 4y^2 - 32x - 8y + 68$

$\therefore 0 = 3x^2 + 3y^2 - 36x + 48 = 0$

$\therefore x^2 + y^2 - 12x + 16 = 0$

(ii) $(x^2 - 12x + 36) + y^2 = 20$

$(x-6)^2 + y^2 = 20$

$\Rightarrow$ centre is $(6, 0)$

& radius $= \underline{\sqrt{20} \text{ units}}$

(c)

Shaded area

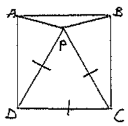
$= \int_0^a f(x)\,dx + \left|\int_a^c f(x)\,dx\right| + \int_c^d f(x)\,dx$

OR

Shaded area

$= \int_0^a f(x)\,dx - \int_a^c f(x)\,dx + \int_c^d f(x)\,dx.$

8 (a)




(i) $\angle ADC = \angle BCD = 90^\circ$ ($\angle$'s of a square)

$\angle PDC = \angle PCD = 60^\circ$ ($\angle$'s of equilateral $\Delta$)

$\therefore \angle ADP = 30^\circ = \angle BCP$ (differences of equal amounts)

$AD = BC$ (sides of a square)

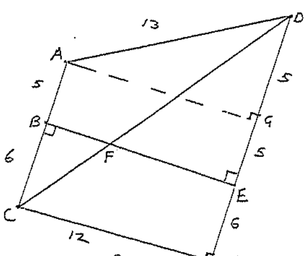
$PD = PC$ (sides of equilateral $\Delta$)

$\therefore \Delta APD \equiv \Delta BPC$ (SAS)

(ii) $AP = BP$ (corresponding sides of congruent $\Delta$'s)

$\therefore \Delta APB$ is isosceles.

(b)




(i) $\angle CBF = 90^\circ = \angle DEF$ (given)

$\angle BFC = \angle DFE$ (vertically opposite $\angle$'s)

$\therefore \Delta BFC \parallel\!\mid \Delta EFD$ (equiangular).

(ii) $AG = \sqrt{13^2 - 5^2}$

$= 12$

$= BE = CH$

$CD = \sqrt{12^2 + 16^2}$

$= 20$

Let $DF = x \Rightarrow CF = 20 - x$

$\therefore \frac{x}{10} = \frac{20-x}{6}$ {corresponding sides of sim. $\Delta$'s are in same ratio}

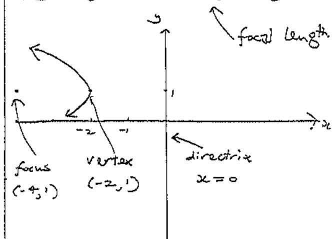
$\therefore 6x = 200 - 10x$

$16x = 200$

$x = 12.5$

$\therefore DF = 12.5$ cm.

(c) $(y-1)^2 = -4(2)(x+2)$




9

(i) $x_A = t + 12 - t^2$

$\dot{x}_A = 1 - 2t$

when $\dot{x}_A = 0$, $1 - 2t = 0$

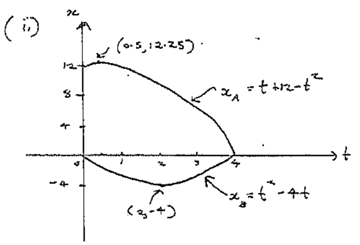
$\therefore t = \frac{1}{2}$

when $t = \frac{1}{2}$, $x_A = \frac{1}{2} + 12 - \frac{1}{4}$

$= 12.25$ m

$\therefore$ A is stat. at $x = 12.25$ m after 0.5 s.

(ii)




$x_A = -t^2 + t + 12$ & $x_B = t^2 - 4t$

$= (4-t)(3+t)$ $= t(t-4)$

(iii) for $0 \le t \le 4$,

$D = x_A - x_B$

$= t + 12 - t^2 - (t^2 - 4t)$

$= t + 12 - t^2 - t^2 + 4t$

$= 5t + 12 - 2t^2$

(iv) $\frac{dD}{dt} = 5 - 4t$

& $\frac{d^2D}{dt^2} = -4 \Rightarrow \frown$

when $\frac{dD}{dt} = 0$, $4t = 5$

i.e. $t = \frac{5}{4}$

given distance function is concave down for all t, D is a maximum at $t = \frac{5}{4}$.

Hence A & B are furthest apart after 1.25 s

(v) $\dot{x}_A = 1 - 2t$ & $\dot{x}_B = 2t - 4$

$\dot{x}_A = \dot{x}_B \Rightarrow 1 - 2t = 2t - 4$

$\therefore 4t = 5$

$\therefore t = 1.25$ s

(vi) $\ddot{x}_A = -2\ ms^{-2}$

$\ddot{x}_B = 2\ ms^{-2}$

Both accelerations are constant and the same magnitude; however A's is positive & B's is negative.

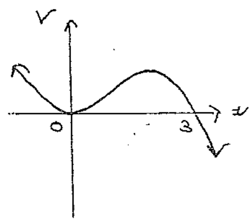
(10)

(a)(i)

$$l + 4x = 12$$
$$\therefore l = 12 - 4x$$
$$V = x \times x \times (12 - 4x)$$
$$= x^2(12 - 4x)$$
$$= 12x^2 - 4x^3$$

(ii)

$$V = 4x^2(3 - x)$$




$$\Rightarrow 0 < x < 3$$

$$\frac{dV}{dx} = 24x - 12x^2$$

$$\frac{d^2V}{dx^2} = 24 - 24x$$

$$\frac{dV}{dx} = 0 \Rightarrow 12x(2 - x)$$
$$\Rightarrow x = 0 \text{ or } x = 2$$
$$\times$$

when $x = 2$, $\frac{d^2V}{dx^2} = 24 - 48$
$$= -24$$
$$< 0 \Rightarrow \text{max.}$$

when $x = 2$, $V = 48 - 32$
$$= 16$$

$\therefore$ Max. vol. $= 16\text{m}^3$

(b)(i) ($\alpha$)

$$A_1 = 400\,000 \times 1.005 - M$$
$$A_2 = 400\,000 \times 1.005^2 - 1.005M - M$$

($\beta$) $A_{300} = 400\,000 \times 1.005^{300}$
$$- M(1 + 1.005^1 + \dots + 1.005^{299})$$

But $A_{300} = 0$, so

$$M = \frac{400\,000 \times 1.005^{300}}{\frac{(1.005^{300} - 1)}{0.005}}$$

$$= 2577.205606$$
$$= \$2577.21 \text{ (n. cent)}$$

(ii) $A_{60} = 400\,000 \times 1.005^{60}$
$$- 2577.21(1 + 1.005^1 + \dots + 1.005^{59})$$
$$= 400\,000 \times 1.005^{60} - 2577.21 \times \frac{1.005^{60} - 1}{0.005}$$
$$= 359\,728.0407$$
$$= \$359\,728.04 \text{ (n. cent)}$$

$$A_{61} = 359728.04 \times 1.0075 - N$$

$$A_{62} = 359728.04 \times 1.0075^2 - 1.0075N - N$$

$\vdots$

$$A_{300} = 359728.04 \times 1.0075^{240}$$
$$- N(1 + 1.0075 + \dots + 1.0075^{239})$$

But $A_{300} = 0$, so

$$N = \frac{359728.04 \times 1.0075^{240}}{\frac{(1.0075^{240} - 1)}{0.0075}}$$

$$= 3236.566546$$

$\therefore$ Req'd am't $= \$3236.57$ (n. cent)

(iii) Extra paid $= \$(3236.57 - 2577.21) \times 240$
$$= \$158\,246.40$$